7.3.1 Exercises

For a link to all of the additional resources available for this section, click OSttS Chapter 7 materials.

In Exercises 1 - 8, sketch the graph of the given parabola. Find the vertex, focus and directrix. Include the endpoints of the latus rectum in your sketch.

For help with these exercises click on one of the resources below:

- Graphing parabolas
- Finding the focus and directrix of a parabola

1. $(x-3)^2 = -16y$
2. $\left(x+\frac{7}{3}\right)^2 = 2\left(y+\frac{5}{2}\right)$
3. $(y-2)^2 = -12(x+3)$
4. $(y+4)^2 = 4x$
5. $(x-1)^2 = 4(y+3)$
6. $(x+2)^2 = -20(y-5)$
7. $(y-4)^2 = 18(x-2)$
8. $\left(y+\frac{3}{2}\right)^2 = -7\left(x+\frac{9}{2}\right)$

In Exercises 9 - 14, put the equation into standard form and identify the vertex, focus and directrix.

For help with these exercises click on one of the resources below:

- Completing the square to put the equation of a parabola in standard form
- Finding the focus and directrix of a parabola

9. $y^2 - 10y - 27x + 133 = 0$
10. $25x^2 + 20x + 5y - 1 = 0$
11. $x^2 + 2x - 8y + 49 = 0$
12. $2y^2 + 4y + x - 8 = 0$
13. $x^2 - 10x + 12y + 1 = 0$
14. $3y^2 - 27y + 4x + \frac{211}{4} = 0$

In Exercises 15 - 18, find an equation for the parabola which fits the given criteria.

For help with these exercises click on the resource below:

- Finding the equation of a parabola

15. Vertex $(7, 0)$, focus $(0, 0)$
16. Focus $(10, 1)$, directrix $x = 5$
17. Vertex $(-8, -9)$; $(0, 0)$ and $(-16, 0)$ are points on the curve
18. The endpoints of latus rectum are $(-2, -7)$ and $(4, -7)$
19. The mirror in Carl's flashlight is a paraboloid of revolution. If the mirror is 5 centimeters in diameter and 2.5 centimeters deep, where should the light bulb be placed so it is at the focus of the mirror?

20. A parabolic Wi-Fi antenna is constructed by taking a flat sheet of metal and bending it into a parabolic shape.[5] If the cross section of the antenna is a parabola which is 45 centimeters wide and 25 centimeters deep, where should the receiver be placed to maximize reception?

21. A parabolic arch is constructed which is 6 feet wide at the base and 9 feet tall in the middle. Find the height of the arch exactly 1 foot in from the base of the arch.

22. A popular novelty item is the 'mirage bowl.' Follow this link to see another startling application of the reflective property of the parabola.

23. With the help of your classmates, research spinning liquid mirrors. To get you started, check out this website.

Checkpoint Quiz 7.3

1. Put $2y^2 + 12y + 8x + 2 = 0$ into standard form and graph. Find the vertex, focus, and directrix.

2. Find the equation of the parabola with focus $(-2, 3)$ and directrix the x-axis.

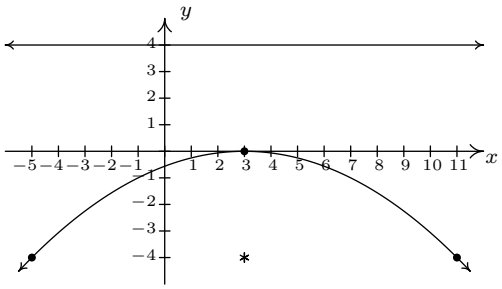
For worked out solutions to this quiz, click the link below:

- Quiz Solution

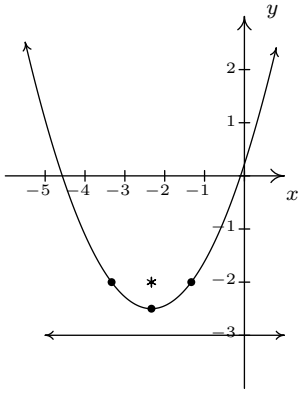
[5] This shape is called a 'parabolic cylinder.'

7.3.2 Answers

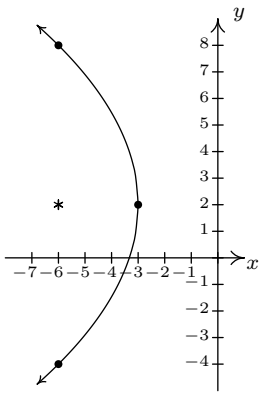
1. $(x-3)^2 = -16y$
   Vertex $(3, 0)$
   Focus $(3, -4)$
   Directrix $y = 4$
   Endpoints of latus rectum $(-5, -4)$, $(11, -4)$



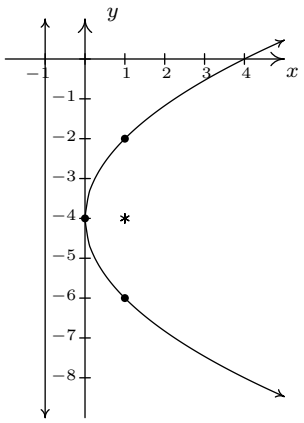
2. $\left(x + \frac{7}{3}\right)^2 = 2\left(y + \frac{5}{2}\right)$
   Vertex $\left(-\frac{7}{3}, -\frac{5}{2}\right)$
   Focus $\left(-\frac{7}{3}, -2\right)$
   Directrix $y = -3$
   Endpoints of latus rectum $\left(-\frac{10}{3}, -2\right)$, $\left(-\frac{4}{3}, -2\right)$



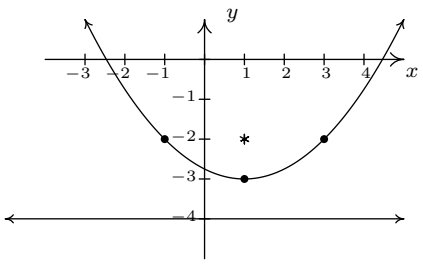
3. $(y-2)^2 = -12(x+3)$
   Vertex $(-3, 2)$
   Focus $(-6, 2)$
   Directrix $x = 0$
   Endpoints of latus rectum $(-6, 8)$, $(-6, -4)$



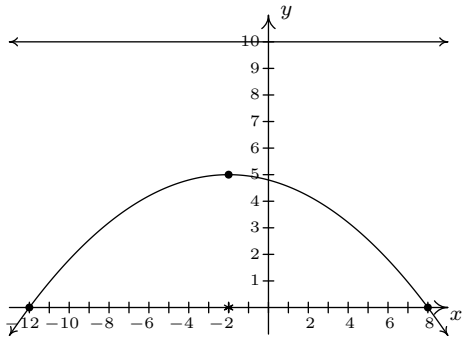
4. $(y+4)^2 = 4x$
   Vertex $(0,-4)$
   Focus $(1,-4)$
   Directrix $x=-1$
   Endpoints of latus rectum $(1,-2)$, $(1,-6)$



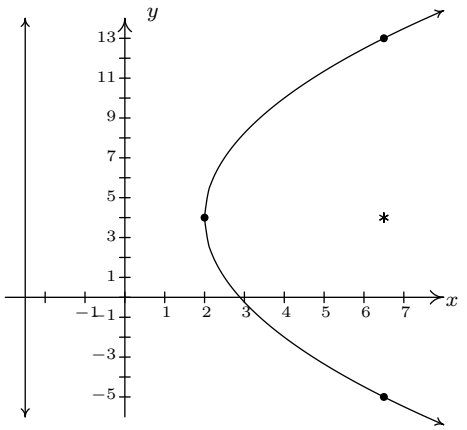
5. $(x-1)^2 = 4(y+3)$
   Vertex $(1,-3)$
   Focus $(1,-2)$
   Directrix $y=-4$
   Endpoints of latus rectum $(3,-2)$, $(-1,-2)$



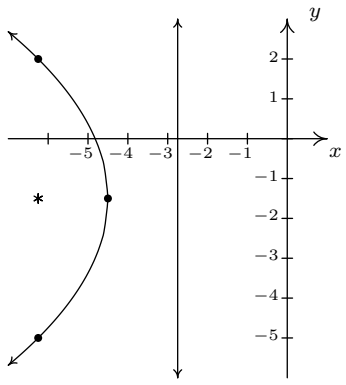
6. $(x+2)^2 = -20(y-5)$
   Vertex $(-2,5)$
   Focus $(-2,0)$
   Directrix $y=10$
   Endpoints of latus rectum $(-12,0)$, $(8,0)$



7. $(y-4)^2 = 18(x-2)$
   Vertex $(2,4)$
   Focus $\left(\frac{13}{2},4\right)$
   Directrix $x=-\frac{5}{2}$
   Endpoints of latus rectum $\left(\frac{13}{2},-5\right)$, $\left(\frac{13}{2},13\right)$



8. $\left(y+\frac{3}{2}\right)^2 = -7\left(x+\frac{9}{2}\right)$
   Vertex $\left(-\frac{9}{2}, -\frac{3}{2}\right)$
   Focus $\left(-\frac{25}{4}, -\frac{3}{2}\right)$
   Directrix $x = -\frac{11}{4}$
   Endpoints of latus rectum $\left(-\frac{25}{4}, 2\right)$, $\left(-\frac{25}{4}, -5\right)$



9. $(y-5)^2 = 27(x-4)$
   Vertex $(4, 5)$
   Focus $\left(\frac{43}{4}, 5\right)$
   Directrix $x = -\frac{11}{4}$

10. $\left(x+\frac{2}{5}\right)^2 = -\frac{1}{5}(y-1)$
    Vertex $\left(-\frac{2}{5}, 1\right)$
    Focus $\left(-\frac{2}{5}, \frac{19}{20}\right)$
    Directrix $y = \frac{21}{20}$

11. $(x+1)^2 = 8(y-6)$
    Vertex $(-1, 6)$
    Focus $(-1, 8)$
    Directrix $y = 4$

12. $(y+1)^2 = -\frac{1}{2}(x-10)$
    Vertex $(10, -1)$
    Focus $\left(\frac{79}{8}, -1\right)$
    Directrix $x = \frac{81}{8}$

13. $(x-5)^2 = -12(y-2)$
    Vertex $(5, 2)$
    Focus $(5, -1)$
    Directrix $y = 5$

14. $\left(y-\frac{9}{2}\right)^2 = -\frac{4}{3}(x-2)$
    Vertex $\left(2, \frac{9}{2}\right)$
    Focus $\left(\frac{5}{3}, \frac{9}{2}\right)$
    Directrix $x = \frac{7}{3}$

15. $y^2 = -28(x-7)$

16. $(y-1)^2 = 10\left(x-\frac{15}{2}\right)$

17. $(x+8)^2 = \frac{64}{9}(y+9)$

18. $(x-1)^2 = 6\left(y+\frac{17}{2}\right)$ or
    $(x-1)^2 = -6\left(y+\frac{11}{2}\right)$

19. The bulb should be placed 0.625 centimeters above the vertex of the mirror. (As verified by Carl himself!)

20. The receiver should be placed 5.0625 centimeters from the vertex of the cross section of the antenna.

21. The arch can be modeled by $x^2 = -(y-9)$ or $y = 9 - x^2$. One foot in from the base of the arch corresponds to either $x = \pm 2$, so the height is $y = 9 - (\pm 2)^2 = 5$ feet.